On Some New Operations in Fuzzy Soft Set And Intuitionistic Fuzzy Soft Set Theory

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Abstract: In this paper, we first point out that several assertions (proposition 3.2.3, proposition 3.2 (iv), (v), (vi) and proposition 4.2 (iv), (v), (vi) in the previous papers by Maji et al. [P. K. Maji, R. Biswas, A. R. Roy The Journal of Fuzzy Mathematics Vol.9, No.3, (2001) 589-602, 677-692] are not true in general, by counter examples. Furthermore, based on the analysis of several operations on Fuzzy Soft Sets (FSS), Intuitionistic Fuzzy Soft Sets (IFSS) introduced in the same paper, we give some new notions such as the restricted intersection, restricted union, restricted difference and the extended intersection of two FSS and two IFSS. Moreover we improve the notion of complement of a FSS and IFSS, and prove that certain De Morgan's laws hold in FSS theory, as well as IFSS theory with respect to these new operations.

Keywords: *Basic Concepts, Fuzzy connectives and aggregation operators, Fuzzy Soft Set(FSS), Intuitionistic Fuzzy Soft Set(IFSS), Applications.*

I. Introduction

Majority of the available mathematical methods utilized for formal modeling, reasoning and computing are crisp, accurate and deterministic in character. But in ground reality, crisp data is not always the part and parcel of the problems encountered in different fields like economics, engineering, social science, medical science, environment etc. As a result, the traditional classical methods cannot be automatically channelized due to their different types of uncertainties which are inherently associated with these problems. There are theories viz. theory of probability, theory of fuzzy sets [11, 13, 14, 15], theory of intuitionistic fuzzy sets [2, 3], theory of vague sets [5], theory of interval mathematics [3], theory of rough sets [12] which are known as mathematical instruments for dealing with uncertainties. But difficulties present in all these theories have been shown by Molodtsov in [10]. The cause of these problems is possibly related to the inadequacy of the parametrization tool of the theories. As a result Molodtsov started the concept of soft set theory as a new mathematical tool for solving the uncertainties which is free from the above difficulties. Presence of vagueness demanded Fuzzy Soft Set (FSS) [6] to come into picture. But satisfactory evaluation of membership values is not always possible because of the insufficiency in the available information (besides the presence of vagueness) situation. Evaluation of non-membership values is also not always possible for the same reason and as a result there exists an indeterministic part upon which hesitation survives. Certainly fuzzy soft set theory is not suitable to solve such problems. In those situations Intuitionistic Fuzzy Soft Set theory (IFSS)[7, 8] may be more applicable. In the present paper we give some new operations in FSS and IFSS Theory and prove that certain De Morgan's laws hold in FSS theory, as well as IFSS theory with respect to these new operations.

II Preliminaries

We recall the definitions of Soft Set, Fuzzy Soft Set (FSS) and Intuitionistic Fuzzy Soft Set (IFSS) with examples. Then we call back to mind, the union and intersection, property of Soft Sets (or, FSS, or, IFSS) over the common universe, complement of a FSS (or an IFSS) and De Morgan's laws in fuzzy set theory, intuitionistic fuzzy set theory.

A. *Soft Sets:* [10]

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the power set of U. Let $A \subset E$. A pair (F,A) is called a soft set over U, where F is a mapping given by,

F : $A \to P(U)$. In other words, a soft set over U is a parameterized family of subsets of the universe U.

B. *Fuzzy Soft Sets :* [6]

Let U be an initial universe set and E be a set of parameters (which are fuzzy words or sentences involving fuzzy words). Let P(U) denotes the set of all fuzzy sets of U. Let $A \subset E$. A pair $(\tilde{F}, A)$ is called a fuzzy soft set (FSS) over U, where $\tilde{F}$ is a mapping given by, $\tilde{F} : A \to P(U)$.

C. *Intuitionistic Fuzzy Soft Sets [7]:*

Let U be an initial universe set and E be a set of parameters. Let P(U) denotes the set of all intuitionistic fuzzy sets of U. Let $A \subset E$. A pair $(\hat{F}, A)$ is called an intuitionistic fuzzy soft set(IFSS) over U, where $\hat{F}$ is a mapping given by, $\hat{F} : A \to P(U)$.

Now we recall some properties of FSS and IFSS.

D. *Union of two Soft Sets (or, FSS or, IFSS) over the common universe :*

Let (F, A) and (G, B) be two Soft Sets (or, FSS or, IFSS) over the common universe U. Then the union of (F, A) and (G, B) over the common universe is the Soft Set (or, FSS or, IFSS) (H, C) where C = $A \cup B$, and $\forall e \varepsilon$ C,

$$H(e) = \begin{cases} F(e), & if\, e\varepsilon(A-B) \\ G(e), & if\ e\varepsilon(B-A) \\ F(e) \cup G(e), & if\ e\varepsilon A \cap B \end{cases}$$

E. *Intersection of two Soft Sets (or, FSS or, IFSS) over the common universe:*

Let (F, A) and (G, B) be two Soft Sets (or, FSS or, IFSS) over the common universe U. Then the intersection of (F, A) and (G, B) over the common universe is the Soft Set (or, FSS or, IFSS) (H, C) where C = $A \cap B$,

and $\forall e \varepsilon$ C, H(e) = F(e) or, G(e), (as both are same set) [for SS and FSS]

and [for IFSS] $\forall e \varepsilon$ C, H(e) = $F(e) \hat{\cap} G(e)$ [where $\hat{\cap}$ denotes the intuitionistic fuzzy intersection of two intuitionistic fuzzy sets.]

F. *Complement of a FSS (or an IFSS) :*

The complement of a fuzzy soft set (or, an IFSS) $(\hat{F}, A)$ is denoted by, $(\hat{F}, A)^C$ and is defined by, $(\hat{F}, A)^C = (\hat{F}^C, \rceil A)$ where $\hat{F}^C : \rceil A \to$ P(U) is a mapping given by $\hat{F}^C(\alpha)$ = fuzzy complement (or, intuitionistic fuzzy complement) of $F(\rceil \alpha), \forall \alpha \varepsilon \rceil A$.

G. *De Morgan's laws in fuzzy set theory [6]:*

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy sets. Then we have the following:

(i) $(\tilde{A} \tilde{\cup} \tilde{B})^C = \tilde{A}^C \cap \tilde{B}^C$

(ii) $(\tilde{A} \tilde{\cap} \tilde{B})^C = \tilde{A}^C \cup \tilde{B}^C$

H. *De Morgan's laws in intuitionistic*

fuzzy set theory[11]:

Let $\hat{A}$ and $\hat{B}$ be two intuitionistic fuzzy sets. Then we have the following:

(i) $(\hat{A} \hat{\cup} \hat{B})^C = \hat{A}^C \hat{\cap} \hat{B}^C$

(ii) $(\hat{A} \hat{\cap} \hat{B})^C = \hat{A}^C \hat{\cup} \hat{B}^C$

III Counterexamples:

We begin this section with a result given by Maji et al. in [6].

Theorem 3.1 (Proposition 3.3 [6])

i) $((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B))^C = (\tilde{F}, A)^C \tilde{\cup} (\tilde{G}, B)^C$

ii) $((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))^C = (\tilde{F}, A)^C \tilde{\cap} (\tilde{G}, B)^C$

The following example can be used to illuminate the incorrectness of Theorem 3.1(i). Moreover, it indicates that (ii) of Theorem 3.1 is also an ambiguous statement.

Example 3.1

Let E be the universe set of parameters and

A = $\{e_1, e_2, e_3\}$, B = $\{e_3, e_4, e_5\}$ be subsets of E (where each parameter is a fuzzy word or, a sentence involving fuzzy words).

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two FSS over the common universe U = $\{s_1, s_2, s_3, s_4, s_5, s_6\}$ such that,

$\tilde{F}(e_1) = \{s_1/.8, s_2/.3\}$,

$\tilde{F}(e_2) = \{s_3/.6, s_4/.9\}$,

$\tilde{F}(e_3) = \{s_2/.7, s_3/.2, s_6/.5\}$,

$\tilde{G}(e_4) = \{s_5/.3\}$,

$\tilde{G}(e_5) = \{s_6/.8\}$,

$\tilde{G}(e_3) = \{s_3/.3, s_4/.8, s_5/.7, s_6/.4\}$,

Let $((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B)) = (\tilde{H}, A \cup B)$ where

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e), & if\ e\varepsilon(A-B) \\ \tilde{G}(e), & if\ e\varepsilon(B-A) \\ \tilde{F}(e) \cup \tilde{G}(e), & if\ e\varepsilon A \cap B \end{cases}$$

Then,

$\tilde{H}(e_1) = \tilde{F}(e_1) = \{s_1/.8, s_2/.3\}$,

$\tilde{H}(e_2) = \tilde{F}(e_2) = \{s_3/.6, s_4/.9\}$,

$\tilde{H}(e_4) = \tilde{G}(e_4) = \{s_5/.3\}$,

$\tilde{H}(e_5) = \tilde{G}(e_5) = \{s_6/.8\}$,

$\tilde{H}(e_3) = \tilde{F}(e_3) \tilde{\cup} \tilde{G}(e_3) = \{s_2/.7, s_3/.3, s_4/.8, s_5/.7, s_6/.5$

Thus for $\rceil e_3 \varepsilon \rceil A \cap \rceil B$, by Definition 3.4 in [1], we have,

$\tilde{H}^C(\rceil e_3)$ = Fuzzy complement of $\tilde{H}(e_3)$

= $\{s_1/1, s_2/.3, s_3/.7, s_4/.2, s_5/.3, s_6/.5\}$

On the other hand,

Let $((\tilde{F}^C, \rceil A) \tilde{\cup} (\tilde{G}^C, \rceil B)) = (\tilde{K}, \rceil A \cup \rceil B)$

where

$$\tilde{K}(\rceil e) = \begin{cases} \tilde{F}^C(\rceil e), & if\ \rceil e\varepsilon(\rceil A - \rceil B) \\ \tilde{G}^C(\rceil e), & if\ \rceil e\varepsilon(\rceil B - \rceil A) \\ \tilde{F}^C(\rceil e) \tilde{\cup} \tilde{G}^C(\rceil e), & if\ \rceil e\varepsilon \rceil A \cap \rceil B \end{cases}$$

Then for $\rceil e_3 \varepsilon \rceil A \cap \rceil B$,

$$\tilde{K}(\rceil e_3) = \tilde{F}^C(\rceil e_3) \tilde{\cup} \tilde{G}^C(\rceil e_3)$$

$= \{s_1/1, s_2/.3, s_3/.8, s_4/1, s_5/1, s_6/.5\} \tilde{\cup} \{s_1/1, s_2/1, s_3/.7, s_4/.2, s_5/.$

$$= \{s_1/1, s_2/1, s_3/.8, s_4/1, s_5/1, s_6/.6\}$$

Clearly we have,

$\tilde{K}(\rceil e_3) \neq \tilde{H}^C(\rceil e_3)$ for $\rceil e_3 \varepsilon \rceil A \cap \rceil B$.

Consequently, we deduce that,

$((\tilde{F}, A) \tilde{\cup} (\tilde{G}, B))^C \neq (\tilde{F}, A)^C \tilde{\cup} (\tilde{G}, B)^C$,

showing that the assertion (i) in Theorem 3.1 is incorrect.

In addition, let us consider the statement (ii) of Theorem 3.1.

Since

$A \cap B = \{e_3\}, \tilde{F}(e_3) = \{s_2/.7, s_3/.2, s_6/.5\}$, and

$\tilde{G}(e_3) = \{s_3/.3, s_4/.8, s_5/.7, s_6/.4\}$,

we immediately have that

$\tilde{F}(e_3) \neq \tilde{G}(e_3)$, and so by Definition 3.10 in [1], $((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))$, simply does not exist.It follows that $((\tilde{F}, A) \tilde{\cap} (\tilde{G}, B))^C$ also does not exist, which make impossible to check the validity of the equality in (ii).

Therefore we conclude that the second statement in Theorem 3.1 is ambiguous.

Remark: In fact the incorrectness of Theorem 3.1 can be found by checking the proof of it (see [6], pp. 599). Suppose that $(\tilde{F},A)\tilde{\cup}(\tilde{G},B)=(\tilde{H},A\cup B)$ and $(\tilde{F},A)^C\tilde{\cup}(\tilde{G},B)^C=(\tilde{K},\rceil A\cup\rceil B)$. The author in [15] claim that

$\tilde{H}^C(\rceil\alpha)$ = fuzzy complement of $\tilde{F}^C(\rceil\alpha)\tilde{\cup}\tilde{G}^C(\rceil\alpha), \forall\alpha\varepsilon\rceil A\cap\rceil B$.

However for any $\rceil\alpha\varepsilon\rceil A\cap\rceil B$, we have that $\tilde{H}^C(\rceil\alpha)$ = fuzzy complement of $\tilde{H}(\alpha)$ = fuzzy complement of $(\tilde{F}(\alpha)\tilde{\cup}\tilde{G}(\alpha))$ = [fuzzy complement of $\tilde{F}(\alpha)$] $\tilde{\cap}$ [fuzzy complement of $\tilde{G}(\alpha)$] = $\tilde{F}^C(\rceil\alpha)\tilde{\cap}\tilde{G}^C(\rceil\alpha)$. Hence $\tilde{H}^C$ should be as follows:

$$\tilde{H}^C(\rceil\alpha)=\begin{cases}\tilde{F}^C(\rceil\alpha), & if\ \rceil\alpha\varepsilon\rceil A-\rceil B\\ \tilde{G}^C(\rceil\alpha), & if\ \rceil\alpha\varepsilon\rceil B-\rceil A\\ \tilde{F}^C(\rceil\alpha)\tilde{\cap}\tilde{G}^C(\rceil\alpha), & if\ \rceil\alpha\varepsilon\rceil A\cap\rceil B\end{cases}$$

But as pointed out in [6],

$$\tilde{K}(\rceil\alpha)=\begin{cases}fuzzy\ complement\ of\ \tilde{F}^C(\rceil\alpha), & if\ \rceil\alpha\varepsilon\rceil A-\rceil B\\ fuzzy\ complement\ of\ \tilde{G}^C(\rceil\alpha), & if\ \rceil\alpha\varepsilon\rceil B-\rceil A\\ fuzzy\ complement\ of\ (\tilde{F}^C(\rceil\alpha)\tilde{\cup}\tilde{G}^C(\rceil\alpha)), & if\ \rceil\alpha\varepsilon\rceil A\cap\rceil B\end{cases}$$

Consequently, we conclude that $\tilde{H}^C$ and $\tilde{K}$ are different in general. This shows that Theorem 3.1 is actually not true. Now we highlight the errors in Proposition 3.2 of [6] which is stated as follows.

Proposition 3.5 (Proposition 3.2 [6]):

(i) $(\tilde{F},A)\tilde{\cup}(\tilde{F},A)=(\tilde{F},A)$

(ii) $(\tilde{F},A)\tilde{\cap}(\tilde{F},A)=(\tilde{F},A)$

(iii) $(\tilde{F},A)\tilde{\cup}\phi=\phi$, where ϕ is the null FSS.

(iv) $(\tilde{F},A)\tilde{\cap}\phi=\phi$

(v) $(\tilde{F},A)\tilde{\cup}\tilde{A}=\tilde{A}$, where $\tilde{A}$ is the absolute FSS.

(vi) $(\tilde{F},A)\tilde{\cap}\tilde{A}=\tilde{A}$

The Assertions (iv),(v) and (vi) in Proposition 3.5 are not correct. The following example illuminates this fact.

Example 3.6

Suppose that there are four dresses in the universe U given by,

U = $\{d_1,d_2,d_3,d_4\}$

Let A = $\{cotton, woollen, synthetic\}$ be the set of parameters showing the material of the dresses.

Let B be the NOT set of the parameter set A.

ie. B = $\rceil A$ = $\{not\ cotton, not\ woollen, not\ synthetic\}$.

Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two FSS over the common universe U, which describe the composition of the dresses. Let

$$(\tilde{F},A)=\{cotton\ dresses=\phi, woollen\ dresses=\phi, synthetic\ dresses=\phi\}[\ where\ \phi\ is\ the\ null\ fuzzy\ set\ of\ U\ ]$$

$$and\ (\tilde{G},B)=\{not\ cotton\ dresses=U, not\ woollen\ dresses, not\ synthetic\ dresses=U\}$$

By Definitions 3.5, 3.6 introduced by Maji et al. [6], we have that $(\tilde{F},A)$ and $(\tilde{G},B)$ are null FSS and absolute FSS respectively.

Now by Definition 3.9 in [6], the union of $(\tilde{F},A)$ and $(\tilde{G},B)$ is a FSS as follows:

$$((\tilde{F},A)\tilde{\cup}(\tilde{G},B))=\{cotton\ dresses=\phi, woollen\ dresses=\phi, synthetic\ dresses=\phi, not\ cotton\ dresses=U, not\ woollen\ dresses=U, not\ synthetic\ dresses=U\}$$

Clearly, $((\tilde{F},A)\tilde{\cup}(\tilde{G},B))\neq(\tilde{G},B)$, which indicates that the Assertion (v) is not true in general.

Moreover, since $A \cap B = \theta$ (null set), $(\tilde{F}, A)\tilde{\cap}(\tilde{G}, B)$ is neither the null FSS $(\tilde{F}, A)$, nor the absolute FSS $(\tilde{G}, B)$. Hence we deduce that the Assertions (iv) and (vi) are incorrect in general.

IV Some New Operations in FSS Theory:

We point out here that the intersection of two FSS introduced in [6] (see Definition 3.10) is not a clearly defined notion, which suffers from many problems.

Note first that if $(\tilde{F}, A)$ and $(\tilde{G}, B)$ are two different FSS, then it is not necessary for these two FSS to have the same subset of U for a particular common parameter say c $\varepsilon A \cap B$, ie., $\tilde{F}(c) \neq \tilde{G}(c)$ in general. Hence the intersection of two fuzzy soft sets as defined in [6] may only be a partial operation on them.

On the contrary, if this kind of intersection must be regarded as a binary operation, then we can deduce from the definition that any two FSS over a common universe must have the same approximation value-set for any common parameter, but this is surely not the case. For a parameter which represents a vague concept such as costly shirts, each person has his / her own opinion and the approximation value-sets given by different persons may be extremely different. There does not exist an objective criteria to fix a standard approximation value-set for a given parameter. In fact, it is also worth noting that the union of two FSS introduced in [6] (see Definition 3.9) implies that two approximation value-sets of a common parameter could be different, while Definition in the section II, subsection E implicity turns to the contrary of this observation. As illustration one may consider the following example.

Example 4.1:

Let E be the universe set of parameters (where each parameter is a fuzzy word or a sentence involving fuzzy words). Let A = $\{e_1, e_2, e_3\}$, B = $\{e_3, e_4\}$ be subsets of E.

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two FSS over the common universe U = $\{s_1, s_2, s_3, s_4, s_5\}$ such that,

$\tilde{F}(e_1) = \{s_1/.8, s_2/.2\}$,

$\tilde{F}(e_2) = \{s_3/.4, s_4/.9\}$,

$\tilde{F}(e_3) = \{s_2/.7, s_3/.2\}$,

$\tilde{G}(e_4) = \{s_5/.6\}$,

$\tilde{G}(e_3) = \{s_3/.5, s_4/.2, s_5/.8\}$.

Let $((\tilde{F}, A)\tilde{\cap}(\tilde{G}, B)) = (\tilde{H}, A \cap B)$ where

$\tilde{H}(e) = \tilde{F}(e)$ or, $\tilde{G}(e), \forall e \varepsilon A \cap B$ (as $\tilde{F}(e)$ and $\tilde{G}(e)$ are same set).

But here $A \cap B = \{e_3\}$ and $\tilde{F}(e_3), \tilde{G}(e_3)$ are two different fuzzy soft sets.

To ensure that the intersection of two fuzzy soft sets free from the above problems, we introduce the following new definition of intersection.

4.2 Extended Intersection : The extended intersection of two fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over a common universe U is the FSS $(\tilde{H}, C)$, where C = $A \cup B$ and $\forall e \varepsilon C$,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e), & if\ e\varepsilon(A-B) \\ \tilde{G}(e), & if\ e\varepsilon(B-A) \\ \tilde{F}(e)\tilde{\cap}\tilde{G}(e), & if\ e\varepsilon A \cap B \end{cases}$$

We write, $((\tilde{F}, A)\tilde{\cap}_E(\tilde{G}, B)) = (\tilde{H}, C)$.

In addition, we may sometimes adopt a different definition of intersection given as follows.

4.3 Restricted Intersection : Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft sets over a common universe U such that $A \cap B \neq \phi$ (null set). The restricted intersection of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ is denoted by $((\tilde{F}, A)\tilde{\cap}_R(\tilde{G}, B))$ and is defined as, $((\tilde{F}, A)\tilde{\cap}_R(\tilde{G}, B)) = (\tilde{H}, C)$, where C = $A \cap B$ and $\forall c \varepsilon C, \tilde{H}(c) = \tilde{F}(c)\tilde{\cap}\tilde{G}(c)$ [where $\tilde{\cap}$ is the operation fuzzy intersection of two fuzzy sets.]

4.5 Restricted Union : Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft sets over a common universe U such that $A \cap B \neq \phi$ (null set). The restricted union of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ is denoted by $((\tilde{F}, A) \tilde{\cup}_R (\tilde{G}, B))$ and is defined as, $((\tilde{F}, A) \tilde{\cup}_R (\tilde{G}, B)) = (\tilde{H}, C)$, where C = $A \cap B$ and $\forall c \varepsilon C, \tilde{H}(c) = \tilde{F}(c) \tilde{\cup} \tilde{G}(c)$ [where $\tilde{\cup}$ is the operation fuzzy union of two fuzzy sets.]

In addition, we may modify and rename the operation union of two intuitionistic fuzzy soft sets over the common universe given by Maji [10], as follows.

4.6 Extended Union :

The extended union of two fuzzy soft sets $(\tilde{F}, A)$ and $(\tilde{G}, B)$ over a common universe U is the FSS $(\tilde{H}, C)$, where C = $A \cup B$ and $\forall e \varepsilon C$,

$$\tilde{H}(e) = \begin{cases} \tilde{F}(e), & if \ e\varepsilon(A - B) \\ \tilde{G}(e), & if \ e\varepsilon(B - A) \\ \tilde{F}(e) \tilde{\cup} \tilde{G}(e), & if \ e\varepsilon A \cap B \end{cases}$$

We write, $((\tilde{F}, A) \tilde{\cup}_E (\tilde{G}, B)) = (\tilde{H}, C)$.

4.8 Restricted Difference :

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft sets over a common universe U such that $A \cap B \neq \phi$ (null set). The restricted difference of $(\tilde{F}, A)$ and $(\tilde{G}, B)$ is denoted by $((\tilde{F}, A) \tilde{-}_R (\tilde{G}, B))$ and is defined as, $((\tilde{F}, A) \tilde{-}_R (\tilde{G}, B)) = (\tilde{K}, P)$, where P = $A \cap B$ and $\forall p \varepsilon P, \tilde{K}(p) = \tilde{F}(p) \tilde{-}_R \tilde{G}(p)$ [by [2] the fuzzy difference of two fuzzy sets $\tilde{F}$ (p) and $\tilde{G}$ (p) is denoted by $\tilde{F}(p) \tilde{-} \tilde{G}(p)$ and is defined as $\tilde{F}(p) \tilde{-} \tilde{G}(p) = \tilde{F}(p) \tilde{\cap} \tilde{G}^C(p)$.]

4.9 Relative Null FSS:

Let U be an initial universe set, E be the universe set of parameters and $A \subset E$. The FSS $(\tilde{F}, A)$ is called a relative null FSS (with respect to the parameter set A), denoted by ϕ_A, if $\tilde{F}$ (e) = null fuzzy set of U, $\forall e \varepsilon A$

4.10 Relative Whole FSS :

Let U be an initial universe set, E be the universe set of parameters and $A \subset E$. The FSS $(\tilde{F}, A)$ is called a relative whole FSS (with respect to the parameter set A), denoted by U_A, if $\tilde{F}(e)$ = U, $\forall e \varepsilon A$. The relative whole FSS U_E with respect to the universe set of parameters E is called the absolute FSS over U.

4.11 Relative Complement of a FSS:

The relative complement of a FSS $(\tilde{F}, A)$ is denoted by $(\tilde{F}, A)^r$ and is defined by,

$(\tilde{F}, A)^r = (\tilde{F}^r, A)$ where $\tilde{F}^r : A \to$ P(U) is a mapping given by, $\tilde{F}^r(\alpha)$ = fuzzy complement of $\tilde{F}(\alpha), \forall \alpha \varepsilon A$.

Consequently, $((\tilde{F}, A)^r)^r = (\tilde{F}, A)$.

It is worth noting that in the above new definition of complement, the parameter set of the complement $((\tilde{F}, A)^r)^r$ is still the original parameter set A, instead of $\rceil A$ as in Definition 3.4 in [10]. To emphasize this difference, the complement given by Definition 3.4 will be called neg-complement (or, pseudo-complement) in what follows.

V De Morgan's laws in FSS theory:

In this section, we first show that the following De Morgan's type of results hold in FSS Theory for the newly defined relative complement, restricted union and restricted intersection.

Theorem 5.1

Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two fuzzy soft sets over a common universe U such that $A \cap B \neq \phi$ Then

i) $((\tilde{F}, A) \tilde{\cup}_R (\tilde{G}, B))^r = (\tilde{F}, A)^r \tilde{\cap}_R (\tilde{G}, B)^r$

ii) $((\tilde{F}, A) \tilde{\cap}_R (\tilde{G}, B))^r = (\tilde{F}, A)^r \tilde{\cup}_R (\tilde{G}, B)^r$

Proof:

i) Let $((\tilde{F}, A) \tilde{\cup}_R (\tilde{G}, B)) = (\tilde{H}, C)$ where

$$\tilde{H}(c) = \tilde{F}(c) \tilde{\cup} \tilde{G}(c), \forall c \varepsilon C = A \cap B \neq \phi$$

Therefore $((\tilde{F},A)\tilde{\cup}_R(\tilde{G},B))^r = (\tilde{H},C)^r$ where

$(\tilde{H},C)^r = (\tilde{H}^r, C)$ where

$\tilde{H}^r(\alpha)$ = fuzzy complement of $\tilde{H}(\alpha), \forall \alpha\varepsilon C.$

$ie.\tilde{H}^r(\alpha) = [\tilde{F}(\alpha)\tilde{\cup}\tilde{G}(\alpha)]^r [\ by (1)]$

$= \tilde{F}^r(\alpha)\tilde{\cap}\tilde{G}^r(\alpha) [\ by\ De\ Morgan's\ laws\ of\ \ fuzzy\ set\ theory\ [13]]$ (2)

On the other hand,

$(\tilde{F},A)^r\tilde{\cap}_R(\tilde{G},B)^r = (\tilde{K},C)$ where C = $A\cap B$ and $\forall\alpha\varepsilon C$,

$\tilde{K}(\alpha) = \tilde{F}^r(\alpha)\tilde{\cap}\tilde{G}^r(\alpha)[\ by\ definition\ ]$

$= \tilde{H}^r(\alpha)[\ by (2)]$

Hence $((\tilde{F},A)\tilde{\cup}_R(\tilde{G},B))^r = (\tilde{F},A)^r\tilde{\cap}_R(\tilde{G},B)^r$

ii)Let $((\tilde{F},A)\tilde{\cap}_R(\tilde{G},B)) = (\tilde{H},C)$ where

$\tilde{H}(\alpha) = \tilde{F}(\alpha)\tilde{\cap}\tilde{G}(\alpha), \forall\alpha\varepsilon C = A\cap B \neq \phi[\ by\ definition\ of\ restricted\ intersection.]$ (3)

Therefore $((\tilde{F},A)\tilde{\cap}_R(\tilde{G},B))^r = (\tilde{H},C)^r = (\tilde{H}^r, C)$ where

$\tilde{H}^r(\alpha) = [\tilde{F}(\alpha)\tilde{\cap}\tilde{G}(\alpha)]^r [\ by\ (3)]$

$= \tilde{F}^r(\alpha)\tilde{\cup}\tilde{G}^r(\alpha) [\ by\ De\ Morgan's\ laws\ of\ \ fuzzy\ set\ theory\ [13]]$ (4)

On the other hand,

$(\tilde{F},A)^r\tilde{\cup}_R(\tilde{G},B)^r = (\tilde{K},C)$ where C = $A\cap B$ and $\forall\alpha\varepsilon C$,

$\tilde{K}(\alpha) = \tilde{F}^r(\alpha)\tilde{\cup}\tilde{G}^r(\alpha)[\ by\ definition\ ]$

$= \tilde{H}^r(\alpha)[\ by\ (4)]$

Hence $((\tilde{F},A)\tilde{\cap}_R(\tilde{G},B))^r = (\tilde{F},A)^r\tilde{\cup}_R(\tilde{G},B)^r$.

By using similar techniques, we can prove that the following De Morgan's laws hold in FSS theory for the extended intersection, the extended union and the neg-complement.

Theorem 5.2

Let $(\tilde{F},A)$ and $(\tilde{G},B)$ be two fuzzy soft sets over a common universe U. Then we have the following:

i) $((\tilde{F},A)\tilde{\cup}_E(\tilde{G},B))^C = (\tilde{F},A)^C\tilde{\cap}_E(\tilde{G},B)^C$

ii) $((\tilde{F},A)\tilde{\cap}_E(\tilde{G},B))^C = (\tilde{F},A)^C\tilde{\cup}_E(\tilde{G},B)^C$

Proof:

i) Let $((\tilde{F},A)\tilde{\cup}_E(\tilde{G},B)) = (\tilde{H},C)$ where C = $A\cup B$ and

$$\tilde{H}(\alpha) = \begin{cases} \tilde{F}(\alpha), & if\ \ \alpha\varepsilon(A-B) \\ \tilde{G}(\alpha), & if\ \ \alpha\varepsilon(B-A) \\ \tilde{F}(\alpha)\tilde{\cup}\tilde{G}(\alpha), & if\ \ \alpha\varepsilon A\cap B \end{cases}$$

Therefore $((\tilde{F},A)\tilde{\cup}_E(\tilde{G},B))^C = (\tilde{H},C)^C = (\tilde{H}^C, \rceil C)$, by definition

where $\rceil C = \rceil A\cup\rceil B$ and

$$\tilde{H}^C(\alpha) = \begin{cases} \tilde{F}^C(\alpha), & if\ \ \alpha\varepsilon(\rceil A-\rceil B) \\ \tilde{G}^C(\alpha), & if\ \ \alpha\varepsilon(\rceil B-\rceil A) \\ \tilde{F}^C(\alpha)\tilde{\cap}\tilde{G}^C(\alpha), & if\ \ \alpha\varepsilon\ \rceil A\cap\rceil B \end{cases}$$

$, by DeMorgan's law of fuzzy sets.] Therefore (\tilde{H}^C, \rceil C) =$

Hence $((\tilde{F},A)\tilde{\cup}_E(\tilde{G},B))^C = (\tilde{F},A)^C\tilde{\cap}_E(\tilde{G},B)^C$.

ii) Let $((\tilde{F},A)\tilde{\cap}_E(\tilde{G},B)) = (\tilde{H},C)$ where C = $A\cup B$ and

$$\tilde{H}(\alpha) = \begin{cases} \tilde{F}(\alpha), & if\ \ \alpha\varepsilon(A-B) \\ \tilde{G}(\alpha), & if\ \ \alpha\varepsilon(B-A) \\ \tilde{F}(\alpha)\tilde{\cap}\tilde{G}(\alpha), & if\ \ \alpha\varepsilon A\cap B \end{cases}$$

Therefore $((F,A)\tilde{\cap}_E(\tilde{G},B))^C = (\tilde{H},C)^C = (\tilde{H}^C, \rceil C)$, by definition

where $\rceil C = \rceil A\cup\rceil B$ and

$$\tilde{H}^C(\alpha) = \begin{cases} \tilde{F}^C(\alpha), & if\ \ \alpha\varepsilon(\rceil A-\rceil B) \\ \tilde{G}^C(\alpha), & if\ \ \alpha\varepsilon(\rceil B-\rceil A) \\ \tilde{F}^C(\alpha)\tilde{\cup}\tilde{G}^C(\alpha), & if\ \ \alpha\varepsilon\ \rceil A\cap\rceil B \end{cases}$$

$by DeMorgan's law of fuzzy sets.] Therefore (\tilde{H}^C, \rceil C) =$

Hence, $((\tilde{F},A)\tilde{\cap}_E(\tilde{G},B))^C = (\tilde{F},A)^C\tilde{\cup}_E(\tilde{G},B)^C$

VI Proposition **(Proposition 4.2 [6]):**

(i) $(\hat{F},A)\hat{\cup}(\hat{F},A)=(\hat{F},A)$

(ii) $(\hat{F},A)\hat{\cap}(\hat{F},A)=(\hat{F},A)$

(iii) $(\hat{F},A)\hat{\cup}\phi=\phi$, where ϕ is the null IFSS.

(iv) $(\hat{F},A)\hat{\cap}\phi=\phi$

(v) $(\hat{F},A)\hat{\cup}\hat{A}=\hat{A}$, where $\hat{A}$ is the absolute IFSS.

(vi) $(\hat{F},A)\hat{\cap}\hat{A}=(\hat{F},A)$

The assertions (iv),(v) and (vi) in proposition 6. are not correct. The following example illuminates this fact.

Example 6.1

Suppose that there are four dresses in the universe U given by, U = $\{d_1,d_2,d_3,d_4\}$.

Let A = $\{cotton, woollen, synthetic\}$ be the set of parameters showing the material of the dresses. Let B be the NOT set of the parameter set A.

ie. B = $\rceil A$ = $\{$ *not cotton*, *not woollen*, *not synthetic* $\}$.

Let $(\hat{F},A)$ and $(\hat{G},B)$ be two intuitionistic fuzzy soft sets over the common universe U, which describe the composition of the dresses. Let

$$(\hat{F},A)=\{\ cotton\ dresses\ =\phi,\ woollen\ dresses\ =\phi,\ synthetic\ dresses\ =\phi\}$$

$$[where\ \phi\ is\ the\ null\ intuitionistic\ \ fuzzy\ set\ of\ U]\ and$$

$$(\hat{G},B)=\{not\ cotton\ dresses=U,\ not\ woollen\ dresses=U,\ not\ synthetic\ dresses=U\}$$

By definitions 4.5, 4.6 introduced by Maji et al. [7], we have that $(\hat{F},A)$ and $(\hat{G},B)$ are null IFSS and absolute IFSS respectively.

Now by Definition 4.9 in [7], the union of $(\hat{F},A)$ and $(\hat{G},B)$ is an IFSS as follow

$$((\hat{F},A)\hat{\cup}(\hat{G},B))=\{\ cotton\ dresses\ =\phi,\ woollen\ dresses\ =\phi,\ synthetic\ dresses\ =\phi,\ not\ cotton\ dresses\ =U,\ not\ wooll$$

Clearly, $((\hat{F},A)\hat{\cup}(\hat{G},B))\neq(\hat{G},B)$, which indicates that the assertion (v) is not true in general.

Moreover, since $A\cap B=\theta$ (null set), $(\hat{F},A)\hat{\cap}(\hat{G},B)$ is neither the null IFSS $(\hat{F},A)$, nor the absolute IFSS $(\hat{G},B)$. Hence we deduce that the assertions (iv) and (vi) are incorrect in general.

VII Some New Operations in IFSS Theory:

We introduce the following new definitions in IFSS Theory.

7.1 Extended Intersection : The extended intersection of two intuitionistic fuzzy soft sets $(\hat{F},A)$ and $(\hat{G},B)$ over a common universe U is the IFSS (H, C), where C = $A\cup B$ and $\forall e\varepsilon C$,

$$H(e)=\begin{cases}\hat{F}(e), & if\ e\varepsilon A-B\\ \hat{G}(e), & if\ \ e\varepsilon(B-A)\\ \hat{F}(e)\hat{\cap}\hat{G}(e), & if\ \ e\varepsilon A\cap B\end{cases}$$

We write, $((\hat{F},A)\hat{\cap}_E(\hat{G},B))=(\hat{H},C)$.

In addition, we may modify and rename the operation intersection of two intuitionistic fuzzy soft sets over the common universe given by Maji [7], as follows.

7.2 Restricted Intersection : Let $(\hat{F},A)$ and $(\hat{G},B)$ be two intuitionistic fuzzy soft sets over a common universe U such that $A\cap B\neq\phi$ (null set). The restricted intersection of $(\hat{F},A)$ and $(\hat{G},B)$ is denoted by $((\hat{F},A)\hat{\cap}_R(\hat{G},B))$ and is defined as, $((\hat{F},A)\hat{\cap}_R(\hat{G},B))=(\hat{H},C)$, where C = $A\cap B$ and $\forall c\varepsilon C,\hat{H}(c)=\hat{F}(c)\hat{\cap}\hat{G}(c)$ [where $\hat{\cap}$ is the operation intuitionistic fuzzy intersection of two intuitionistic fuzzy sets.]

7.3 Restricted Union : Let $(\hat{F},A)$ and $(\hat{G},B)$ be two intuitionistic fuzzy soft sets over a common universe U such that $A\cap B\neq\phi$ (null set). The restricted union of $(\hat{F},A)$ and $(\hat{G},B)$ is denoted by $((\hat{F},A)\hat{\cup}_R(\hat{G},B))$ and is defined as, $((\hat{F},A)\hat{\cup}_R(\hat{G},B))=(\hat{H},C)$, where C = $A\cap B$ and $\forall c\varepsilon C,\hat{H}(c)=\hat{F}(c)\hat{\cup}\hat{G}(c)$ [where $\hat{\cup}$ is the operation intuitionistic fuzzy union of two intuitionistic fuzzy sets.]

In addition, we may modify and rename the operation union of two intuitionistic fuzzy soft sets over the common universe given by Maji [7], as follows.

7.4 Extended Union : The extended union of two intuitionistic fuzzy soft sets $(\hat{F}, A)$ and $(\hat{G}, B)$ over a common universe U is the IFSS $(\hat{H}, C)$, where C = $A \cup B$ and $\forall e \varepsilon C$,

$$\hat{H}(e) = \begin{cases} \hat{F}(e), & if\; e\varepsilon(A-B) \\ \hat{G}(e), & if\;\; e\varepsilon(B-A) \\ \hat{F}(e) \hat{\cup} \hat{G}(e), & if\;\; e\varepsilon A \cap B \end{cases}$$

We write, $((\hat{F}, A) \hat{\cup}_E (\hat{G}, B)) = (\hat{H}, C)$

7.5 Restricted Difference : Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U such that $A \cap B \neq \phi$ (null set). The restricted difference of $(\hat{F}, A)$ and $(\hat{G}, B)$ is denoted by $((\hat{F}, A) \smile_R (\hat{G}, B))$ and is defined as, $((\hat{F}, A) \smile_R (\hat{G}, B)) = (\hat{K}, P)$, where P = $A \cap B$ and $\forall p \varepsilon P, \hat{K}(p) = \hat{F}(p) \smile \hat{G}(p)$ [the intuitionistic fuzzy difference of two intuitionistic fuzzy sets $\hat{F}(p)$ and $\hat{G}(p)$ is denoted by $\hat{F}(p) \smile \hat{G}(p)$ and is defined as $\hat{F}(p) \smile \hat{G}(p) = \hat{F}(p) \hat{\cap} \hat{G}^C(p)$.]

7.6 Relative Null IFSS: Let U be an initial universe set, E be the universe set of parameters and $A \subset E$. The IFSS $(\hat{F}, A)$ is called a relative null IFSS (with respect to the parameter set A), denoted by ϕ_A, if $\hat{F}$ (e) = null intuitionistic fuzzy set of U, $\forall e \varepsilon A$

7.7 Relative Whole IFSS: Let U be an initial universe set, E be the universe set of parameters and $A \subset E$. The IFSS $(\hat{F}, A)$ is called a relative whole IFSS (with respect to the parameter set A), denoted by U_A, if $\hat{F}$ (e) = U, $\forall e \varepsilon A$. The relative whole IFSS U_E with respect to the universe set of parameters E is called the absolute IFSS over U.

7.8 Relative Complement of a IFSS: The relative complement of a IFSS $(\hat{F}, A)$ is denoted by $(\hat{F}, A)^r$ and is defined by, $(\hat{F}, A)^r = (\hat{F}^r, A)$ where $\hat{F}^r : A \to$ P(U) is a mapping given by,

$\hat{F}^r(\alpha)$ = intuitionistic fuzzy complement of $\hat{F}(\alpha), \forall \alpha \varepsilon A$.

Consequently, $((\hat{F}, A)^r)^r = (\hat{F}, A)$

It is worth noting that in the above new definition of complement, the parameter set of the complement $((\hat{F}, A)^r)^r$ is still the original parameter set A, instead of $\rceil A$ as in Definition 4.4 in [7]. To emphasize this difference, the complement given by Definition 4.4 will be called neg-complement (or, pseudo-complement) in what follows.

VIII De Morgan's laws in IFSS theory:

In this section, we first show that the following De Morgan's type of results hold in IFSS Theory for the newly defined relative complement, restricted union and restricted intersection.

Theorem 8.1

Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U such that $A \cap B \neq \phi$. Then

i) $((\hat{F}, A) \hat{\cup}_R (\hat{G}, B))^r = (\hat{F}, A)^r \hat{\cap}_R (\hat{G}, B)^r$

ii) $((\hat{F}, A) \hat{\cap}_R (\hat{G}, B))^r = (\hat{F}, A)^r \hat{\cup}_R (\hat{G}, B)^r$

Theorem 8.2

Let $(\hat{F}, A)$ and $(\hat{G}, B)$ be two intuitionistic fuzzy soft sets over a common universe U. Then we have the following:

i) $((\hat{F}, A) \hat{\cup}_E (\hat{G}, B))^C = (\hat{F}, A)^C \hat{\cap}_E (\hat{G}, B)^C$

ii) $((\hat{F}, A) \hat{\cap}_E (\hat{G}, B))^C = (\hat{F}, A)^C \hat{\cup}_E (\hat{G}, B)^C$

IX Conclusion:

Ali et al.[1] makes some comments on Maji's paper and offers some new operations in soft sets over the common universe and proves that certain De Morgan's laws hold in Soft set theory with respect to these new operations. In this paper we first point out that some propositions in the previous papers by Maji et al. [6,7] are not true in general, by counter examples. Furthermore considering the fact that the parameters (which are words or sentences) are mostly fuzzy, in this paper we define the same operations for fuzzy soft sets and intuitionistic fuzzy soft sets with appropriate numerical examples. Then we prove that certain De Morgan's laws hold in Fuzzy Soft set theory and intuitionistic fuzzy soft theory with respect to these new operations.These properties may be used in real life problems, like decision making problem, inventory Control problem, etc.

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