Instantaneous Frequency Signature for Estimating the Direction of Arrivals

SedighehGhofrani

Electrical and Electronics Engineering Department

Islamic Azad University, Tehran South Branch

Tehran, IRAN

E-mails: s_ghofrani@azad.ac.ir

***Abstract*—Time-frequency analysis was combined with array processing to develop a direction of arrival (DOA) estimation method.Spatial time-frequency distribution (STFD) was introduced as the natural means to deal with source signals that are localizable in the time-frequency (TF) domain. It was shown that estimating the signal and noise subspaces are improved by constructing the subspaces from the TF signatures of the signal arrivals rather than from the spatial data covariance matrix, which is commonly used in conventional multiple signal classification (MUSIC). Although the STFD overcomes the two problems of being sources non-stationary and low signal to noise ratio (SNR), the fundamental problem with this approach remains the need for the incorporation of STFD matrices computed only at the source auto term points. In other words, the instantaneous frequency (IF) signature is needed in real application of STFD. Identification of auto-term regions are often difficult for really closed space sources. Because the cross term masks the auto terms, it means the cross term amplitude is greater than the auto terms. In this paper, we have done this job by using the matching pursuit (MP) decomposition based on two different type dictionaries, Gaussian and chirplet. The MP distribution is used as a mask in order to extract the true TF points those belong to the signal IF signature or auto-term. Therefore, the auto-term regions are correctly obtained without having any pre-knowledge about the signal source and they are used for constructing the STFD matrix for DOA estimation. The experimental results show that our proposed method outperforms the conventional MUSIC.**

Keywords: Direction of arrival estimation, Instantaneous frequency signature.

I. INTRODUCTION

Antenna arrays collect multidimensional data that contains signals arriving from different sources.The most common structure is the uniform linear array (ULA), which consists of adjacent antenna elements are equally spaced on a straight line by a given distance. In array signal processing, the goal is to extract important information about the originating signals, of which only a mixture is observed. This information may be the number of sources, the DOA estimationof the sources or the signal waveforms. DOA estimation is one of the most fundamental tasks in array processing in order to find the spatial location of the impinging signals.The most well-known algorithm that reconstructs the spatial covariance matrix and works based on signal and noise subspace is MUSIC[1]. Under the condition that the observation period or the number of snapshot is long and SNR is not too low, this approach is known as a high resolution and accurate method which is widely used in the design of smart antennas[2]-[3].

In many scenarios, signalsare non-stationary (i.e. the spectrumsare time variant such as the frequency modulatedsignals, LFM) and close in space. Since the non-stationary signals exhibit time varying spectra, TFdistributions especially those belong to the Cohen class provide a natural means for the analysis of such signals[4]-[5]. STFD were introduced as the natural means to deal with source signals that are localizable in the TF domain [6]-[9].The STFD framework applies a form of joint-variable signal representation to expose any hidden TF signatures characterizing the data received by the antenna array. Signal analysis in a single domain, whether time or frequency, fails to reveal the local behavior of the signal and in expressing its power distribution over both time and frequency. On the other hand, bilinear transforms, such as Cohen's class [4]-[5] of time-frequency distributions (TFD), capture the IF laws underlying the non-stationarity of the data. The STFD matrix, in lieu of the spatial covariance matrix, permits the auto- and cross-TFDs of the sensor data to retain the signal phase and, as such, embeds the source's DOA information. DOA estimation approaches using subspace methods, such as MUSIC [1], and incorporating the STFDs have been shown to improve the performance over their covariance matrix counterparts[10], primarilybecause of their capability to successfully discriminate among sources and exclude some from consideration prior to subspace decomposition. Accordingly, the STFD-based DOA approaches become attractive for sources with close angular separations, but with distinctive IFs [11].

The main challenge for STFD implementation is finding the correct TF points those are located on the true signal IF signature whenever source signals are close and so the cross terms magnitude is significant in comparison with the auto terms. So, it is known that the main difficulty of applying the

STFD-based DOA estimation approaches is the existence of cross-termsin the presence of multi-component signals [12].

MP is an adaptive signal decomposition technique [13]. It is energy conservative and was introduced based on using the Gabor functions as TF atoms at first and has been extended to other dictionaries, such as the chirplet dictionary [14]-[15]. Clearly, using atoms with more parameters provides higher flexibility in matching the signal, but also increases the computational cost. As long as, there is a compatibility between the decomposed signal and the used dictionary type, the MP distribution or auto-WVD is cross term free [16].

In this paper we employ the MP based on Gaussian and chirplet atoms to exploit the IF signature and implement the STFD for DOA estimation. The main advantages of using the MP decomposition is robust against cross terms and additive Gaussian noise as well. The experimental results shows that our proposed method outperforms the conventional MUSIC.

This paper is organized as follows. In Section II, we review Wigner Ville distribution (WVD) as the best well known bilinear TF distribution belongs to the Cohen class, the conventional MUSIC, and STFD technique for DOA estimation. In Section III, at first, we provide a short overview of MP in general when the signal is noise free and also explain the MP distribution based on both Gabor and chirplet dictionaries, then, we present our proposed method for DOA estimation where the MP helps STFD and find the signal IF signature, we also show the experimental results. Finally, in Section IV, we have conclusion.

The following notations are used in this paper. Boldface lower-case letters (e.g., $\mathbf{a}$) denote vectors, and boldface upper-case letters (e.g., $\mathbf{A}$) denote matrices. $E[.]$ represents the statistical mean operation. $(.)^*$, $(.)^T$ and $(.)^H$ denote complex conjugate, transpose and conjugate transpose, respectively. $\delta(.)$ denotes the Kronecker delta function, and $\mathbf{I}$ is an identity matrix. .

II. PRELIMINARY

In this section, we review the WVDas the best wellknown distribution belong to the Cohen class,the MUSIC algorithm as the conventional method, and STFD as the modern method based on bilinear TF distribution for DOA estimation.

A. Wigner Ville Distribution

Time-frequency distributions have been used extensively for non-stationary signal analysis. The distribution describes the energy density of a signal simultaneously in both time and frequency. The class of all quadratic, shift invariant TF is known as the Cohen class [4]-[5]. WVD is the well-known belongs to this class:

$$W_x(t,\omega) = \int_{-\infty}^{+\infty} x(t+\frac{\tau}{2})\, x^*(t-\frac{\tau}{2})\, e^{-j\omega\tau} d\tau \tag{1}$$

where $\omega = 2\pi f$. WVD has the best resolution among all other TF distributions in the Cohen class, but it generates cross-terms interference when analyzing multi-component signals. These cross terms show energy which does not really exist at particular time/frequency co-ordinates. The existence of cross-terms also causes the WV distribution to present negative values in certain regions of the TF plane and thereby renders the physical interpretation problematic. It is important to note that there are two types of cross terms named as inner- and outer-interference result of the interactions between the components of the same source signal, which is the case when the source signal itself is of multi-components and interactions between two signal components belonging to two different sources.

B. MUSIC

Assuming there are K non-coherent narrow-band signal sources with the different impinging angle $\theta_k, k = 1,2,..., K$ in the space, the array is a ULA with M array elements. The array output vector is,

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{2}$$

where $t = 1,2,...,N$, and N is the number of snapshots, $\mathbf{x}(t) = [x_1(t), x_2(t),..., x_M(t)]^T$ is the array output vector, $\mathbf{s}(t) = [s_1(t), s_2(t),..., s_K(t)]^T$ is the signal source vector and $\mathbf{n}(t) = [n_1(t), n_2(t),..., n_M(t)]^T$ is an additive noise vector whose elements are modeled as stationary, spatially and temporally white Gaussian, zero-mean complex random processes, independent of the source signals. That is, $E[\mathbf{n}(t+\tau)\mathbf{n}^H(t)] = \sigma_n^2 \delta(\tau)\mathbf{I}$, where σ_n^2 is the variance. The k-th column of the mixing matrix $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2),..., \mathbf{a}(\theta_K)]$ is expressed as $\mathbf{a}(\theta_k) = [1, e^{j\omega_k},... e^{j(M-1)\omega_k}]^T$, where $\omega_k = 2\pi \frac{d}{\lambda}\sin(\theta_k)$ is the spatial frequency, λ denotes the wavelength, and d is the inter-element spacing. The output array covariance matrix or the spatial covariance matrix is:

$$\mathbf{C_{xx}} = E[\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}\mathbf{C_{ss}}\mathbf{A}^H + \sigma_n^2\mathbf{I} \tag{3}$$

where $\mathbf{C_{ss}} = E[\mathbf{s}(t)\mathbf{s}^H(t)]$ is the source covariance matrix. When the number of sources is less than the number of sensors, the source DOAs can be estimated by employing subspace approach, MUSIC [1]. The basic idea of the MUSIC algorithm is projecting the search steering vector to the noise subspace, $\mathbf{E_n}$, of the spatial covariance matrix $\mathbf{C_{xx}}$. In practice, the true covariance matrix $\mathbf{C_{xx}}$ is unknown, and is estimated from the available data vectors as $\hat{\mathbf{C}}_{\mathbf{xx}} = \sum_{t=1}^{N} \mathbf{x}(t)\mathbf{x}^H(t) / N$. Denote the estimate of the noise subspace as $\hat{\mathbf{E}}_{\mathbf{n}}$, then the DOA of the sources in the field of view is estimated by searching the peaks ofthe spatial pseudo-spectrum:

$$P(\theta) = \frac{1}{\mathbf{a}^H(\theta)\hat{\mathbf{E}}_{\mathbf{n}}\hat{\mathbf{E}}_{\mathbf{n}}^H\mathbf{a}(\theta)} \tag{4}$$

C. Spatial Time Frequency Distribution

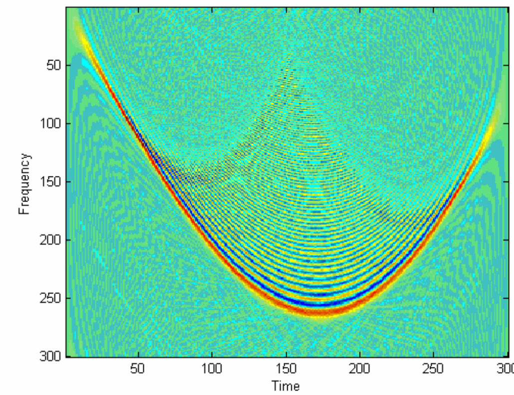
Under the linear data model, (2), the STFD matrix can be defined for any Cohen's class of TFDs [6]-[7]. For WVD, the statistical expectation of the STFD matrix of $\mathbf{x}(t)$ is expressed as,

$$E[\mathbf{W_{xx}}(t,\omega)] = \mathbf{A}E[\mathbf{W_{ss}}(t,\omega)]\mathbf{A}^H + \sigma_n^2\mathbf{I} \qquad (5)$$

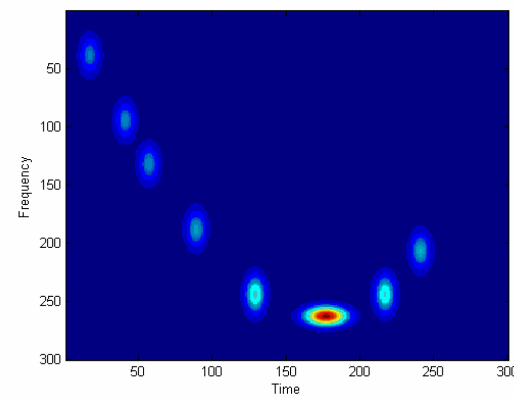
where $\mathbf{W_{ss}}(t,\omega)$ is the signal TFD matrix whose entries are the auto- and cross-source WVDs. The dimension of matrix $\mathbf{W_{xx}}(t,\omega)$ is $M \times M$, whereas that of $\mathbf{W_{ss}}(t,\omega)$ is $K \times K$. We note that $E[\mathbf{W_{xx}}(t,\omega)]$ can be constructed for any selected TF points or TF regions. The spatially averaged TFD matrix was proposed in [8]-[9] to reduce noise and cross-terms contributions. It was shown [6]-[7] that the formula relating the TF distribution matrix of the sensor data to that of the sources, (5), is identical to the relationship between the data covariance matrix and the source correlation matrix, (3), when the additive noise is spatially and temporally white Gaussian, zero-mean complex random processes, and independent of the source signals. So, the TF correlation matrix, $\hat{C}_{xx}$,

$$\hat{C}_{xx} = \begin{bmatrix} W_{x_1x_1} & W_{x_1x_2} & \cdots & W_{x_1x_M} \\ W_{x_2x_1} & W_{x_2x_2} & \cdots & W_{x_2x_M} \\ \vdots & \vdots & \cdots & \vdots \\ W_{x_Mx_1} & W_{x_Mx_2} & \cdots & W_{x_Mx_M} \end{bmatrix} \qquad (6)$$

is constructed by averaging on proper TF points or regions, then the eigen decomposition to $\hat{C}_{xx}$ can be performed to identify the signal and noise subspace and then estimate the DOAs. The proper TF points are those belong to the sources auto-terms.As a result of the averaging process is decreasing the noise level. Any way the main difficulty of the above approach is need to construct the STFD matrix, (6), from auto term points. On the other word, we need the signal IF signature. In this paper by using the MP decomposition based on Gaussian and chirplet, we define a proper mask in order to find the auto terms or proper TF points and construct the STFD matrixor the TF correlation matrix by averaging without having any pre-knowledge about signal sources.



(a)



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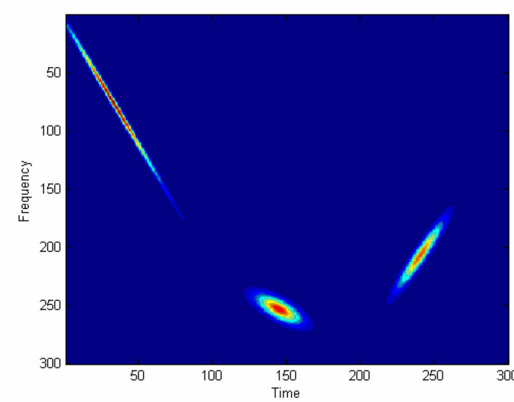


(c)

Figure 1. Time frequency distribution of polynomial phase signal, (a) WVD, (b) MP distribution based on Gaussian atom (decomposition number is 8), (c) MP distribution based on chirplet atom (decomposition number is 3).

III. DOA ESTIMATION

In this section, we provide a short overview of MP in general based on both Gabor and chirplet dictionaries. Then, we present our proposed method for DOA estimation where the MP helps STFD and find the signal IF signature, we alsoperform some simulations in order to have a comparison with the conventional MUSIC.

A. MP Decomposition

The MP adaptive signal decompositionis based on a dictionary that contains a family of functions called elementary functions or TF atoms [13].Let $D = \{\mathbf{g}_l\}$ is a redundant dictionary including a family of functions which are normalized to unit norm, i.e. $\|\mathbf{g}_l\| = 1$, where $\|g_l\|^2 = <\mathbf{g}_l, \mathbf{g}_l> = \sum_t g_l(t) g_l^*(t)$ denotes the vector inner product, and 't' refers to discrete time. The decomposition of a signal is performed by projecting the signal over the function dictionary and then selecting the atoms which can best match the local structure of the signal. So, we compute a linear expansion of $\mathbf{x}$ over a set of elementary functions selected from the dictionary in order to best match its inner structures. After L iterations, the MP decomposition of an arbitrary signal $\mathbf{x}$can be written as,

$$\mathbf{x} = \sum_{l=0}^{L-1} \tilde{c}_l \, \mathbf{g}_l + R^L \mathbf{x} \,, \tag{7}$$

where $R^L \mathbf{x}$ is the residue after L times signal decomposition, $\mathbf{g}_l$ is the chosen atom, l is the iteration index, and $\tilde{c}_l$ is the complex coefficient,

$$\tilde{c}_l = < R^l \mathbf{x}, \mathbf{g}_l > \tag{8}$$

which is obtained by the inner product of the residual $R^l \mathbf{x}$ and atom $\mathbf{g}_l$. By letting, $R^0 \mathbf{x} = \mathbf{x}$, the MP algorithm decomposes the residue at each stage. It was shown [13] that the MP recovers the components of any noise free signal, $\mathbf{x}$, when the dictionary is complete,

$$\mathbf{x} = \sum_{l=0}^{+\infty} \tilde{c}_l \, \mathbf{g}_l \tag{9}$$

In this case, the MP decomposition is energy conservative [13], i.e.,

$$\|\mathbf{x}\|^2 = \sum_{l=0}^{+\infty} |\tilde{c}_l|^2 \tag{10}$$

Since the equality in (9) applies to all signal samples, $t = 1, 2, ..., N$, it can be written as,

$$x(t) = \sum_{l=0}^{+\infty} \tilde{c}_l \, g_l(t) \,. \tag{11}$$

The MP dictionary includes TF atoms, $g(t) = \frac{1}{\sqrt{s}} \gamma(\frac{t-u}{s}) e^{j\phi(t)}$, in general, where $\gamma(t) = 2^{1/4} e^{-\pi t^2}$ denotes the Gaussian envelope and $\phi(t)$ is an arbitrary phase. Although MP at first introduced with Gabor dictionary [13], where $\phi(t) = \zeta\, t$ and the atoms are specified by three parameters, (s, u, ζ), it has been extended to other dictionaries, such as the chirplet dictionary [14]-[15], where $\phi(t) = \zeta\, t + \frac{\beta}{2} t^2$ and the atoms are defined by four parameters, (s, u, ζ, β). We notice, among all atoms with Gaussian shape envelope, the Gabor and chirplet atoms are unique in the sense that they have the highest concentration in both the time and frequency domains [17]. The elements of the parameter set (s, u, ζ, β) are real, and denote, in order, the width, the time center, the frequency center, and the frequency modulation rate. In addition, the width, s, is always positive. Generally, there is no analytical solution for finding the optimum values of MP atoms parameters. As a result, there is a trade-off between flexibility in matching the signal and

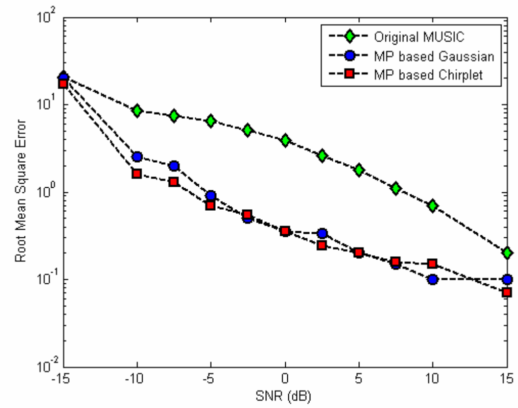
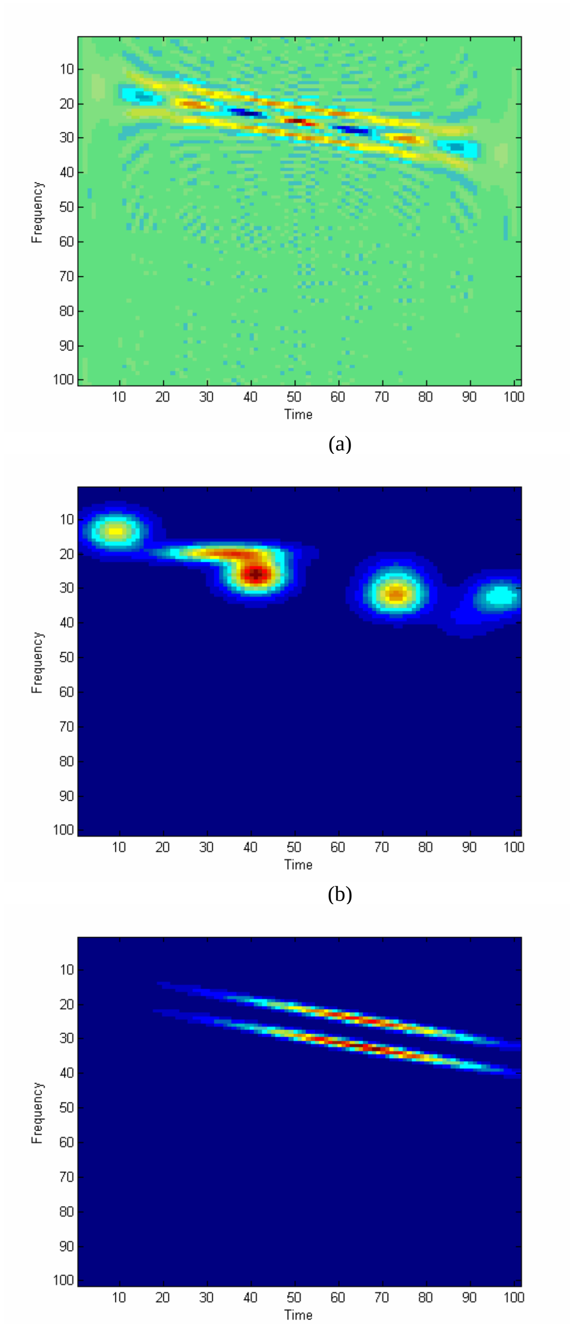


Figure 2. RMSE of DOA estimation for original MUSIC, and our proposed method. The impinging angle is 10° and the number of trial is 20, the number of snapshot is 301.

computational cost. It is noted that the MP attempts to decompose a discrete time signal into a large number of atoms which are generated from a mother Gauusian function, with a pre-considered phase based on the dictionary type. The atoms are sampled in time to have the same length, N, as input signal or the residuals. In addition, the parameters that described an atom should also be discretized. In this paper, according to the length of observed signal snapshots, we discretize the parameter s, followed by other variables, to generate the dictionary based on Gabor, and chirplet[18]. A high value of M and a zero value for the residual energy decompose a signal completely, (10), and The WVD of the decomposed signal is,

$$W_x(t,\omega) = \sum_{l=0}^{+\infty} |\tilde{c}_l|^2 \, W_{g_l}(t,\omega) + \sum_{l=0}^{+\infty} \sum_{\substack{l'=0 \\ l \neq l'}}^{+\infty} \tilde{c}_l \tilde{c}_{l'}^* W_{g_l g_{l'}}(t,\omega) \tag{12}$$

The double sum corresponds to the cross terms of the WVD, so the MP distribution is defined by keeping the first term [13],



(a) (b) (c)

Figure 3. TF distribution of two parallel LFM signal, (a) WVD, (b) MP distribution based on Gaussian atom (decomposition number is 8), (c) MP distribution based on chirplet atom (decomposition number is 2).

$$E_x(t,\omega)=\sum_{l=0}^{+\infty}|\tilde{c}_l|^2\, W_{g_l}(t,\omega). \tag{13}$$

where $W_{g_{\gamma l}}(t,\omega)$ is the WVD of the appropriate atoms. As long as the chosen atoms are either Gaussian or chirplet, then the MP distribution is always positive irrespective of the signal being analysed and it can be interpreted as an energy density function of x in the TF plane. In [19], we derived the MP distribution based on these two dictionaries as follows:

$$E_x(t,\omega)=2\sum_{l=0}^{+\infty}|\tilde{c}_l|^2\, H\left(\frac{t-u_l}{s_l}\right)F(s_l(\omega-\zeta_l)) \tag{14}$$

$$E_x(t,\omega)=2\sum_{l=0}^{+\infty}|\tilde{c}_l|^2\, H\left(\frac{t-u_l}{s_l}\right)F(s_l(\omega-\zeta_l-\beta_l(t-u_l))) \tag{15}$$

where $H(t)=e^{-2\pi t^2}$; $F(\omega)=e^{-\frac{1}{2\pi}\omega^2}$.

B. Our Proposed Method and Experimental Results

MUSIC algorithm is not suitable for signal sources which are non-stationary and close in space. Making use of TF transformation can overcome the limitations of conventional subspace method, MUSIC, but the STFD matrix should be constructed by choosing the proper TF points those belong to the signal source auto terms. Adaptive signal decomposition or MP is an approach to avoid cross terms in the TF plane and thus have no problematic physical interpretation in comparison with the other distributions belong to the Cohen class. In this paper we use the MP distribution (because of mentioned feature) based on using both Gaussian and chirplet atoms in order to obtain a proper mask for averaging in order to implement the STFD and thereby obtaining the TF correlation matrix. For this purpose, at first we decompose only the first output array that is considered as the reference element. Then we compute the MP distribution and we sure no cross terms are there. Finally, we use the MP distribution as a mask in order to find the proper TF points those belong to the sources auto terms and obtain the TF correlation matrix by averaging on those selected TF points or regions for eigen decomposition and so DOAs estimation. In this work, no needs to distinguish the IF source signature. Now in following, during two simulation examples, we show, how our proposed algorithm outperforms the conventional MUSIC where as it choose the proper TF points automatically.

Example 1: The ULA of M=4 sensors with equal inter-sensor spacing of half wavelength ($d=\lambda/2$) is used, and N=301 snapshots are employed for each simulation run. We consider the polynomial phase signal as a source

$$s(t)=\exp(\,j2\pi(0.317\,t\sin(0.002\,\pi t))) \tag{16}$$

with 10° impinge on the array from the far-field. The WVD of the source is shown in Fig. 1-a. Although the signal seems to be one component, there is inner-interference, because of being signal polynomial phase. We decompose the signal of reference element (chose 8 Gaussian atoms and 3 chirplet atoms) and the MP distribution based on Gaussian and chirplet

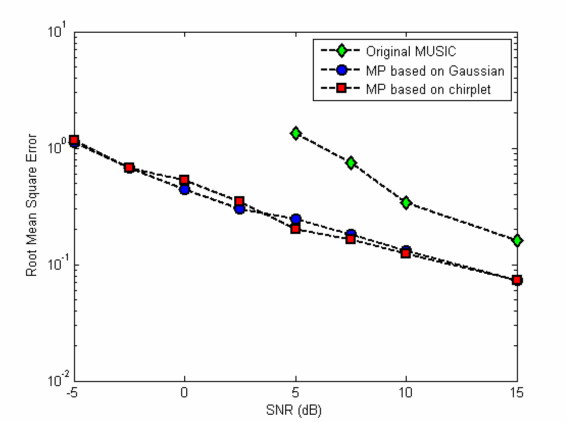


Figure 4. RMSE of DOA estimation for original MUSIC, and our proposed method. The impinging angle is 10° and 20° and the number of trial is 20, the number of snapshot is 101.

atoms are shown in Figs. 1-b and 1-c. The noise is additive Gaussian white noise. The results are computed over 20 independent runs for each SNR under the same condition. Fig. 2 shows the averaged root mean square error (RMSE) of DOA estimation of our proposed method and the conventional MUSIC. Although there is only one source, because of being non-stationary, the performance of our proposed algorithm outperforms the conventional MUSIC.

Example 2: The ULA of M=8 sensors with equal inter-sensor spacing of half wavelength ($d = \lambda/2$) is used, and N=101 snapshots are employed for each simulation run. We consider two LFM signal sources,

$$s_1(t) = \exp(j2\pi(9t + 2t^2)),\ s_2(t) = \exp(j2\pi(4t + 2t^2)) \qquad (17)$$

with impinge on the array from the far-field. The incident directions are 10° and 20°. The WVD of the two sources is shown in Fig. 3-a. For single LFM signal, WV distribution can gain quite high resolution both in time domain and frequency domain. However, it has a very big drawback that it will generate cross interference terms in case of multi component signals, which will seriously influence the DOA estimation performance reduced. Although some people try to eliminate the cross interference terms to improve the DOA estimation accuracy in case of multi signal sources, we try to detect and use only the auto terms by MP.As the two parallel LFM are really close, the cross term amplitude is greater than auto-terms. It means the methods based on finding peaks fail. We decompose the signal of reference element (chose 8 Gaussian atom and 2 chirplet atom) and obtain the MP distributions which are shown in Figs. 3-b and 3-c. The noise is additive Gaussian white noise. The MP distribution is used as a mask in order to construct the STFD matrix by averaging TF points which belong to signal IF signature or auto terms. The RMSE of DOA estimation are computed over 20 independent runs for each SNR for our proposed method and conventional MUSIC and shown in Fig. 4. Because the spectrum of the source signal is time-variant, the conventional MUSIC failed to resolve the two closely spaced sources at low SNR.

Although our proposed method based on both Gaussian and chirplet has close performance, using the chirplet atoms has more freedom and thereby can intuitively better match the signal under consideration with different IF signature. As a consequence, the MP algorithm running time is problematic.

IV. CONCLUSION

It was shown that for non-stationary and closely space signals as uncorrelated sources in array processing, the MUSIC algorithm failed to estimate the DOAs especially for low SNR. Constructing the STFD matrix and then eigen value decomposition is one way to overcome this problem, but for implementation, the IF signature is needed. In this paper, A new method to exploit the IF signature is proposed by using the adaptive MP decomposition based on two different dictionaries, the Gaussian and chirplet. By using the MP adaptive signal decomposition, the auto terms are detected automatically and no concern about existing cross terms are. The result indicates, the algorithm successfully estimates the direction of arrival of the incidents signals impinging on the antenna array and outperforms the conventional MUSIC.

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AUTHOR PROFILE

SedighehGhofrani was born 1968 in Ghochan, recieved BSc degree in Electronic Engineering from Tehran university, Iran, in 1991, the MSc. Degree in communication from Islamic Azad university, south Tehran branch, Iran, in 1997 and Ph.D. in Electronic from Iran university of science and technology, in 2004. She has been the assistant professor of Electrical Engineering Department at the Islamic Azad university, south Tehran branch since 2004. Her area of research includes image processing and signal processing. In 2003, she spent eight months at the School of Electronic and Electrical Engineering, the University of Leeds, UK, supported by British Council foundation. In 2012, she spent eight months at the Center for Advanced Communications at Villanova University, PA, USA, as visiting research professor.